

Parallel Universes

Max Tegmark

Dept. of Physics, Univ. of Pennsylvania, Philadelphia, PA 19104; max@physics.upenn.edu
(January 23 2003.)

Abstract: I survey physics theories involving parallel universes, which form a natural four-level hierarchy of multiverses allowing progressively greater diversity. Level I: A generic prediction of inflation is an infinite ergodic universe, which contains Hubble volumes realizing all initial conditions — including an identical copy of you about $10^{10^{29}}$ m away. Level II: In chaotic inflation, other thermalized regions may have different physical constants, dimensionality and particle content. Level III: In unitary quantum mechanics, other branches of the wavefunction add nothing qualitatively new, which is ironic given that this level has historically been the most controversial. Level IV: Other mathematical structures give different fundamental equations of physics. The key question is not whether parallel universes exist (Level I is the uncontroversial cosmological concordance model), but how many levels there are. I discuss how multiverse models can be falsified and argue that there is a severe “measure problem” that must be solved to make testable predictions at levels II-IV.

Is there another copy of you reading this article, deciding to put it aside without finishing this sentence while you are reading on? A person living on a planet called Earth, with misty mountains, fertile fields and sprawling cities, in a solar system with eight other planets. The life of this person has been identical to yours in every respect – until now, that is, when your decision to read on signals that your two lives are diverging.

You probably find this idea strange and implausible, and I must confess that this is my gut reaction too. Yet it looks like we will just have to live with it, since the simplest and most popular cosmological model today predicts that this person actually exists in a Galaxy about $10^{10^{29}}$ meters from here. This does not even assume speculative modern physics, merely that space is infinite and rather uniformly filled with matter as indicated by recent astronomical observations. Your *alter ego* is simply a prediction of the so-called concordance model of cosmology, which agrees with all current observational evidence and is used as the basis for most calculations and simulations presented at cosmology conferences. In contrast, alternatives such as a fractal universe, a closed universe and a multiply connected universe have been seriously challenged by observations.

The farthest you can observe is the distance that light has been able to travel during the 14 billion years since the big-bang expansion began. The most distant visible objects are now about 4×10^{26} meters away*, and a sphere

of this radius defines our observable universe, also called our *Hubble volume*, our *horizon volume* or simply our universe. Likewise, the universe of your above-mentioned twin is a sphere of the same size centered over there, none of which we can see or have any causal contact with yet. This is the simplest (but far from the only) example of parallel universes.

By this very definition of “universe”, one might expect the notion that our observable universe is merely a small part of a larger “multiverse” to be forever in the domain of metaphysics. Yet the epistemological borderline between physics and metaphysics is defined by whether a theory is experimentally testable, not by whether it is weird or involves unobservable entities. Technology-powered experimental breakthroughs have therefore expanded the frontiers of physics to incorporate ever more abstract (and at the time counterintuitive) concepts such as a round rotating Earth, an electromagnetic field, time slowdown at high speeds, quantum superpositions, curved space and black holes. As reviewed in this article, it is becoming increasingly clear that multiverse models grounded in modern physics can in fact be empirically testable, predictive and falsifiable. Indeed, as many as four distinct types of parallel universes (Figure 1) have been discussed in the recent scientific literature, so that the key question is not whether there is a multiverse (since Level I is rather uncontroversial), but rather how many levels it has.

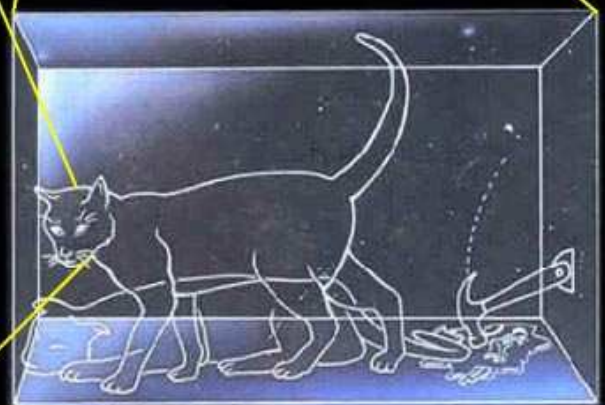
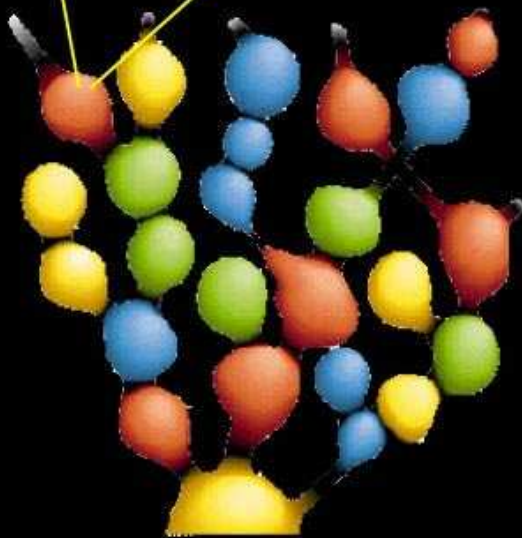
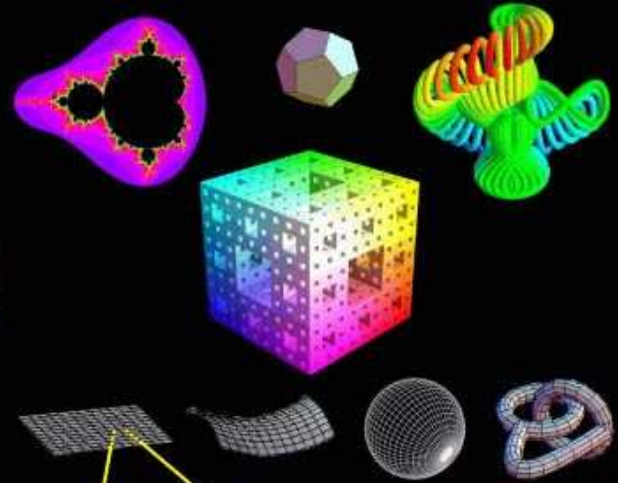
*After emitting the light that is now reaching us, the most distant things we can see have receded because of the cosmic expansion, and are now about about 40 billion light years away.

Level 1: Regions beyond our cosmic horizon

Features: Same laws of physics, different initial conditions
Assumptions: Infinite space, ergodic matter distribution
Evidence: - Microwave background measurements point to flat, infinite space, large-scale smoothness
- Simplest model

Level 4: Other mathematical structures

Features: Different fundamental equations of physics
Assumption: Mathematical existence = physical existence
Evidence: - Unreasonable effectiveness of math in physics
- Answers Wheeler/Hawking question: "why these equations, not others"



Level 2: Other post-inflation bubbles

Features: Same fundamental equations of physics, but perhaps different constants, particles and dimensionality
Assumption: Chaotic inflation occurred
Evidence: - Inflation theory explains flat space, scale-invariant fluctuations, solves horizon problem and monopole problems and can naturally explain such bubbles
- Explains fine-tuned parameters

Level 3: The Many Worlds of Quantum Physics

Features: Same as level 2
Assumption: Physics unitary
Evidence: - Experimental support for unitary physics
- AdS/CFT correspondence suggests that even quantum gravity is unitary
- Decoherence experimentally verified
- Mathematically simplest model

I. LEVEL I: REGIONS BEYOND OUR COSMIC HORIZON

Let us return to your distant twin. If space is infinite and the distribution of matter is sufficiently uniform on large scales, then even the most unlikely events must take place somewhere. In particular, there are infinitely many other inhabited planets, including not just one but infinitely many with people with the same appearance, name and memories as you. Indeed, there are infinitely many other regions the size of our observable universe, where every possible cosmic history is played out. This is the Level I multiverse.

A. Evidence for Level I parallel universes

Although the implications may seem crazy and counter-intuitive, this spatially infinite cosmological model is in fact the simplest and most popular one on the market today. It is part of the cosmological concordance model, which agrees with all current observational evidence and is used as the basis for most calculations and simulations presented at cosmology conferences. In contrast, alternatives such as a fractal universe, a closed universe and a multiply connected universe have been seriously challenged by observations. Yet the Level I multiverse idea has been controversial (indeed, an assertion along these lines was one of the heresies for which the Vatican had Giordano Bruno burned at the stake in 1600[†]), so let us review the status of the two assumptions (infinite space and “sufficiently uniform” distribution).

How large is space? Observationally, the lower bound has grown dramatically (Figure 2) with no indication of an upper bound. We all accept the existence of things that we cannot see but could see if we moved or waited, like ships beyond the horizon. Objects beyond cosmic horizon have similar status, since the observable universe grows by a light-year every year as light from further away has time to reach us[‡]. Since we are all taught about simple Euclidean space in school, it can therefore be difficult to imagine how space could *not* be infinite — for what would lie beyond the sign saying “*SPACE ENDS HERE — MIND THE GAP*”? Yet Einstein’s theory of gravity allows space to be finite by being differently connected than Euclidean space, say with the topology of

a four-dimensional sphere or a doughnut so that traveling far in one direction could bring you back from the opposite direction. The cosmic microwave background allows sensitive tests of such finite models, but has so far produced no support for them — flat infinite models fit the data fine and strong limits have been placed on both spatial curvature and multiply connected topologies. In addition, a spatially infinite universe is a generic prediction of the cosmological theory of inflation (Garriga & Vilenkin 2001b). The striking successes of inflation listed below therefore lend further support to the idea that space is after all simple and infinite just as we learned in school.

How uniform is the matter distribution on large scales? In an “island universe” model where space is infinite but all the matter is confined to a finite region, almost all members of the Level I multiverse would be dead, consisting of nothing but empty space. Such models have been popular historically, originally with the island being Earth and the celestial objects visible to the naked eye, and in the early 20th century with the island being the known part of the Milky Way Galaxy. Another non-uniform alternative is a fractal universe, where the matter distribution is self-similar and all coherent structures in the cosmic galaxy distribution are merely a small part of even larger coherent structures. The island and fractal universe models have both been demolished by recent observations as reviewed in Tegmark (2002). Maps of the three-dimensional galaxy distribution have shown that the spectacular large-scale structure observed (galaxy groups, clusters, superclusters, *etc.*) gives way to dull uniformity on large scales, with no coherent structures larger than about 10^{24} m. More quantitatively, imagine placing a sphere of radius R at various random locations, measuring how much mass M is enclosed each time, and computing the variation between the measurements as quantified by their standard deviation ΔM . The relative fluctuations $\Delta M/M$ have been measured to be of order unity on the scale $R \sim 3 \times 10^{23}$ m, and dropping on larger scales. The Sloan Digital Sky Survey has found $\Delta M/M$ as small as 1% on the scale $R \sim 10^{25}$ m and cosmic microwave background measurements have established that the trend towards uniformity continues all the way out to the edge of our observable universe ($R \sim 10^{27}$ m), where $\Delta M/M \sim 10^{-5}$. Barring conspiracy theories where the universe is designed to fool us, the observations thus speak loud and clear: space as we know it continues far beyond the edge of our observable universe, teeming with galaxies, stars and planets.

B. What are Level I parallel universes like?

The physics description of the world is traditionally split into two parts: initial conditions and laws of physics specifying how the initial conditions evolve. Observers

[†]Bruno’s ideas have since been elaborated by, *e.g.*, Brundrit (1979), Garriga & Vilenkin (2001b) and Ellis (2002), all of whom have thus far avoided the stake.

[‡]If the cosmic expansion continues to accelerate (currently an open question), the observable universe will eventually stop growing.

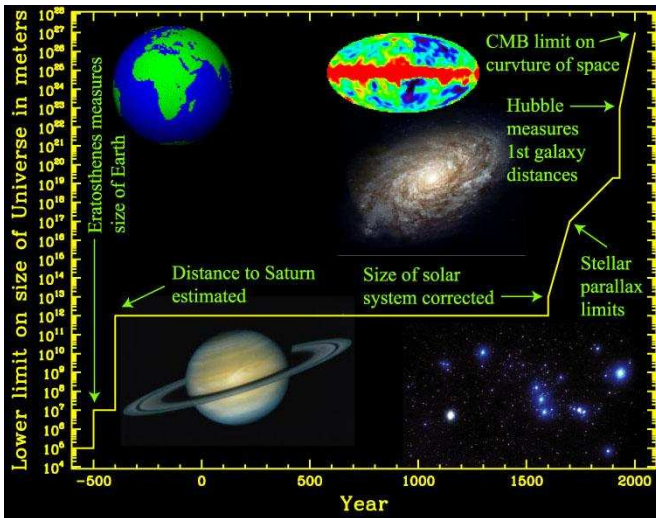


FIG. 2. Although an infinite universe has always been a possibility, the lower limit on the size of our universe has kept growing.

living in parallel universes at Level I observe the exact same laws of physics as we do, but with different initial conditions than those in our Hubble volume. The currently favored theory is that the initial conditions (the densities and motions of different types of matter early on) were created by quantum fluctuations during the inflation epoch (see section 3). This quantum mechanism generates initial conditions that are for all practical purposes random, producing density fluctuations described by what mathematicians call an ergodic random field.[§] *Ergodic* means that if you imagine generating an ensemble of universes, each with its own random initial conditions, then the probability distribution of outcomes in a given volume is identical to the distribution that you get by sampling different volumes in a single universe. In other words, it means that everything that could in principle have happened here did in fact happen somewhere else.

Inflation in fact generates all possible initial conditions with non-zero probability, the most likely ones being almost uniform with fluctuations at the 10^{-5} level that are amplified by gravitational clustering to form galaxies,

[§]Strictly speaking, the random field is ergodic if 1) Space is infinite, 2) the mass fluctuations $\Delta M/M$ approach zero on large scales (as measurements suggest), and 3) the densities at any set of points has a multivariate Gaussian probability distribution (as predicted by the most popular inflation models, which can be traced back to the fact that the harmonic oscillator equation governing the inflaton field fluctuations gives a Gaussian wavefunction for the ground state). For the technical reader, conditions 2 and 3 can be replaced by the weaker requirement that correlation functions of all order vanish in the limit of infinite spatial separation.

stars, planets and other structures. This means both that pretty much all imaginable matter configurations occur in some Hubble volume far away, and also that we should expect our own Hubble volume to be a fairly typical one — at least typical among those that contain observers. A crude estimate suggests that the closest identical copy of you is about $\sim 10^{10^{29}}$ m away. About $\sim 10^{10^{91}}$ m away, there should be a sphere of radius 100 light-years identical to the one centered here, so all perceptions that we have during the next century will be identical to those of our counterparts over there. About $\sim 10^{10^{115}}$ m away, there should be an entire Hubble volume identical to ours.**

This raises an interesting philosophical point that will come back and haunt us in Section VB: if there are indeed many copies of “you” with identical past lives and memories, you would not be able to compute your own future even if you had complete knowledge of the entire state of the cosmos! The reason is that there is no way for you to determine which of these copies is “you” (they all feel that they are). Yet their lives will typically begin to differ eventually, so the best you can do is predict probabilities for what you will experience from now on. This kills the traditional notion of determinism.

C. How a multiverse theory can be tested and falsified

Is a multiverse theory one of metaphysics rather than physics? As emphasized by Karl Popper, the distinction between the two is whether the theory is empirically testable and falsifiable. Containing unobservable entities does clearly *not* per se make a theory non-testable. For instance, a theory stating that there are 666 parallel universes, all of which are devoid of oxygen makes the testable prediction that we should observe no oxygen here, and is therefore ruled out by observation.

As a more serious example, the Level I multiverse framework is routinely used to rule out theories in mod-

**This is an extremely conservative estimate, simply counting all possible quantum states that a Hubble volume can have that are no hotter than 10^8 K. 10^{115} is roughly the number of protons that the Pauli exclusion principle would allow you to pack into a Hubble volume at this temperature (our own Hubble volume contains only about 10^{80} protons). Each of these 10^{115} slots can be either occupied or unoccupied, giving $N = 2^{10^{115}} \sim 10^{10^{115}}$ possibilities, so the expected distance to the nearest identical Hubble volume is $N^{1/3} \sim 10^{10^{115}}$ Hubble radii $\sim 10^{10^{115}}$ meters. Your nearest copy is likely to be much closer than $10^{10^{29}}$ meters, since the planet formation and evolutionary processes that have tipped the odds in your favor are at work everywhere. There are probably at least 10^{20} habitable planets in our own Hubble volume alone.

ern cosmology, although this is rarely spelled out explicitly. For instance, cosmic microwave background (CMB) observations have recently shown that space has almost no curvature. Hot and cold spots in CMB maps have a characteristic size that depends on the curvature of space, and the observed spots appear too large to be consistent with the previously popular “open universe” model. However, the average spot size randomly varies slightly from one Hubble volume to another, so it is important to be statistically rigorous. When cosmologists say that the open universe model is ruled out at 99.9% confidence, they really mean that if the open universe model were true, then fewer than one out of every thousand Hubble volumes would show CMB spots as large as those we observe — therefore the entire model with all its infinitely many Hubble volumes is ruled out, even though we have of course only mapped the CMB in our own particular Hubble volume.

The lesson to learn from this example is that multiverse theories *can* be tested and falsified, but only if they predict what the ensemble of parallel universes is and specify a probability distribution (or more generally what mathematicians call a *measure*) over it. As we will see in Section V B, this measure problem can be quite serious and is still unsolved for some multiverse theories.

II. LEVEL II: OTHER POST-INFLATION BUBBLES

If you felt that the Level I multiverse was large and hard to stomach, try imagining an infinite set of distinct ones (each symbolized by a bubble in Figure 1), some perhaps with different dimensionality and different physical constants. This is what is predicted by the the currently popular chaotic theory of inflation, and we will refer to it as the Level II multiverse. These other domains are more than infinitely far away in the sense that you would never get there even if you traveled at the speed of light forever. The reason is that the space between our Level I multiverse and its neighbors is still undergoing inflation, which keeps stretching it out and creating more volume faster than you can travel through it. In contrast, you could travel to an arbitrarily distant Level I universe if you were patient and the cosmic expansion decelerates.^{††}

^{††}Astronomical evidence suggests that the cosmic expansion is currently accelerating. If this acceleration continues, then even the level I parallel universes will remain forever separate, with the intervening space stretching faster than light can travel through it. The jury is still out, however, with popular models predicting that the universe will eventually stop accelerating and perhaps even recollapse.

A. Evidence for Level II parallel universes

By the 1970’s, the Big Bang model had proved a highly successful explanation of most of the history of our universe. It had explained how a primordial fireball expanded and cooled, synthesized Helium and other light elements during the first few minutes, became transparent after 400,000 years releasing the cosmic microwave background radiation, and gradually got clumpier due to gravitational clustering, producing galaxies, stars and planets. Yet disturbing questions remained about what happened in the very beginning. Did something appear from nothing? Where are all the superheavy particles known as magnetic monopoles that particle physics predicts should be created early on (the “monopole problem”)? Why is space so big, so old and so flat, when generic initial conditions predict curvature to grow over time and the density to approach either zero or infinity after of order 10^{-42} seconds (the “flatness problem”)? What conspiracy caused the CMB temperature to be nearly identical in regions of space that have never been in causal contact (the “horizon problem”)? What mechanism generated the 10^{-5} level seed fluctuations out of which all structure grew?

A process known as *inflation* can solve all these problems in one fell swoop (see reviews by Guth & Steinhardt 1984 and Linde 1994), and has therefore emerged as the most popular theory of what happened very early on. Inflation is a rapid stretching of space, diluting away monopoles and other debris, making space flat and uniform like the surface of an expanding balloon, and stretching quantum vacuum fluctuations into macroscopically large density fluctuations that can seed galaxy formation. Since its inception, inflation has passed additional tests: CMB observations have found space to be extremely flat and have measured the seed fluctuations to have an approximately scale-invariant spectrum without a substantial gravity wave component, all in perfect agreement with inflationary predictions.

Inflation is a general phenomenon that occurs in a wide class of theories of elementary particles. In the popular model known as *chaotic inflation*, inflation ends in some regions of space allowing life as we know it, whereas quantum fluctuations cause other regions of space to inflate even faster. In essence, one inflating bubble sprouts other inflationary bubbles, which in turn produce others in a never-ending chain reaction (Figure 1, lower left, with time increasing upwards). The bubbles where inflation has ended are the elements of the Level II multiverse. Each such bubble is infinite in size^{‡‡}, yet there are in-

^{‡‡}Surprisingly, it has been shown that inflation can produce an infinite Level I multiverse even in a bubble of finite spatial

finitely many bubbles since the chain reaction never ends. Indeed, if this exponential growth of the number of bubbles has been going on forever, there will be an uncountable infinity of such parallel universes (the same infinity as that assigned to the set of real numbers, say, which is larger than that of the [countably infinite] set of integers). In this case, there is also no beginning of time and no absolute Big Bang: there is, was and always will be an infinite number of inflating bubbles and post-inflationary regions like the one we inhabit, forming a fractal pattern.

B. What are Level II parallel universes like?

The prevailing view is that the physics we observe today is merely a low-energy limit of a much more symmetric theory that manifests itself at extremely high temperatures. This underlying fundamental theory may be 11-dimensional, supersymmetric and involving a grand unification of the four fundamental forces of nature. A common feature in such theories is that the potential energy of the field(s) driving inflation has several different minima (sometimes called “vacuum states”), corresponding to different ways of breaking this symmetry and, as a result, to different low-energy physics. For instance, all but three spatial dimensions could be curled up (“compactified”), resulting in an effectively three-dimensional space like ours, or fewer could curl up leaving a 7-dimensional space. The quantum fluctuations driving chaotic inflation could cause different symmetry breaking in different bubbles, resulting in different members of the Level II multiverse having different dimensionality. Many symmetries observed in particle physics also result from the specific way in which symmetry is broken, so there could be Level II parallel universes where there are, say, two rather than three generations of quarks.

In addition to such discrete properties as dimensionality and fundamental particles, our universe is characterized by a set of dimensionless numbers known as *physical constants*. Examples include the electron/proton mass ratio $m_p/m_e \approx 1836$ and the cosmological constant, which appears to be about 10^{-123} in so-called Planck units. There are models where also such continuous parameters can vary from one one post-inflationary bubble to another.^{§§}

volume, thanks to an effect whereby the spatial directions of spacetime curve towards the (infinite) time direction (Bucher & Spergel 1999).

^{§§}Although the fundamental equations of physics are the same throughout the Level II multiverse, the approximate effective equations governing the low-energy world that we observe will differ. For instance, moving from a three-dimensional to a four-dimensional (non-compactified) space

The Level II multiverse is therefore likely to be more diverse than the Level I multiverse, containing domains where not only the initial conditions differ, but perhaps the dimensionality, the elementary particles and the physical constants differ as well.

Before moving on, let us briefly comment on a few closely related multiverse notions. First of all, if one Level II multiverse can exist, eternally self-reproducing in a fractal pattern, then there may well be infinitely many other Level II multiverses that are completely disconnected. However, this variant appears to be untestable, since it would neither add any qualitatively different worlds nor alter the probability distribution for their properties. All possible initial conditions and symmetry breakings are already realized within each one.

An idea proposed by Tolman and Wheeler and recently elaborated by Steinhardt & Turok (2002) is that the (Level I) multiverse is cyclic, going through an infinite series of Big Bangs. If it exists, the ensemble of such incarnations would also form a multiverse, arguably with a diversity similar to that of Level II.

An idea proposed by Smolin (1997) involves an ensemble similar in diversity to that of Level II, but mutating and sprouting new universes through black holes rather than during inflation. This predicts a form of a natural selection favoring universes with maximal black hole production.

In braneworld scenarios, another 3-dimensional world could be quite literally parallel to ours, merely offset in a higher dimension. However, it is unclear whether such a world (“brane”) deserves to be called a parallel universe separate from our own, since we may be able to interact with it gravitationally much as we do with dark matter.

C. Fine-tuning and selection effects

Physicists dislike unexplained coincidences. Indeed, they interpret them as evidence that models are ruled out. In Section I C, we saw how the open universe model was ruled out at 99.9% confidence because it implies that the observed pattern of CMB fluctuations is extremely unlikely, a one-in-a thousand coincidence occurring in only 0.1% of all Hubble volumes.

changes the observed gravitational force equation from an inverse square law to an inverse cube law. Likewise, breaking the underlying symmetries of particle physics differently will change the lineup of elementary particles and the effective equations that describe them. However, we will reserve the terms “different equations” and “different laws of physics” for the Level IV multiverse, where it is the fundamental rather than effective equations that change.

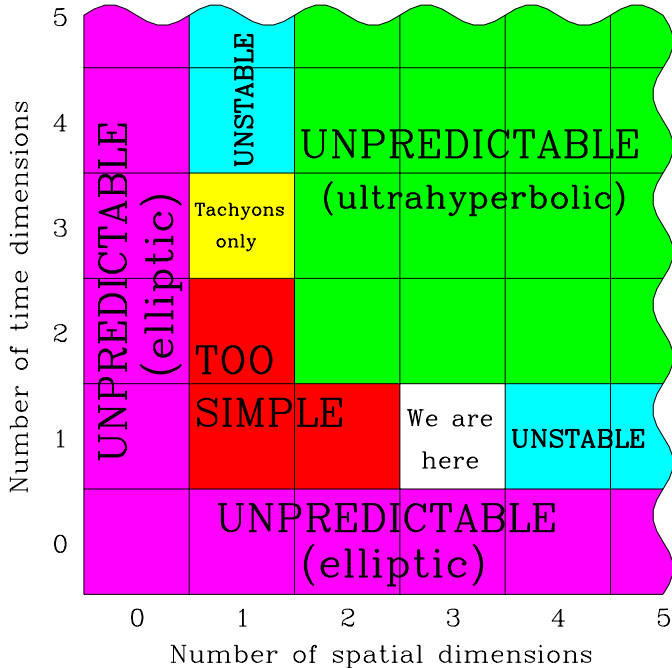


FIG. 3. Why we should not be surprised to find ourselves living in 3+1-dimensional spacetime. When the partial differential equations of nature are elliptic or ultrahyperbolic, physics has no predictive power for an observer. In the remaining (hyperbolic) cases, $n > 3$ admits no stable atoms and $n < 3$ may lack sufficient complexity for observers (no gravitational attraction, topological problems). From Tegmark (1997).

Suppose you check into a hotel, are assigned room 1967 and, surprised, note that that this is the year you were born. After a moment of reflection, you conclude that this is not all that surprising after all, given that the hotel has many rooms and that you would not be having these thoughts in the first place if you'd been assigned another one. You then realize that even if you knew nothing about hotels, you could have inferred the existence of other hotel rooms, because if there were only one room number in the entire universe, you would be left with an unexplained coincidence.

As a more pertinent example, consider M , the mass of the Sun. M affects the luminosity of the Sun, and using basic physics, one can compute that life as we know it on Earth is only possible if M is in the narrow range $1.6 \times 10^{30}\text{kg} - 2.4 \times 10^{30}\text{kg}$ — otherwise Earth's climate would be colder than on Mars or hotter than on Venus. The measured value is $M \sim 2.0 \times 10^{30}\text{kg}$. This apparent coincidence of the habitable and observed M -values may appear disturbing given that calculations show that stars in the much broader mass range $M \sim 10^{29}\text{kg} - 10^{32}\text{kg}$ can exist. However, just as in the hotel example, we can explain this apparent coincidence if there is an ensemble and a selection effect: if there are in fact many solar systems with a range of sizes of the central star and the planetary orbits, then we obviously expect to find ourselves living in one of the inhabitable ones.

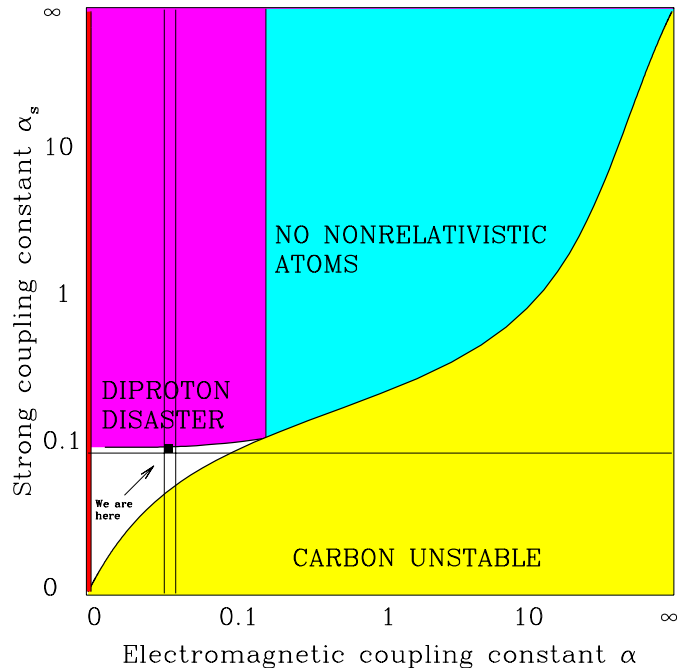


FIG. 4. Hints of fine-tuning for the parameters α and α_s which determine the strengths of the electromagnetic force and the strong nuclear force, respectively (from Tegmark 1997). The observed values $(\alpha, \alpha_s) \approx (1/137, 0.1)$ are indicated with a filled square. Grand unified theories rule out everything except the narrow strip between the two vertical lines, and deuterium becomes unstable below the horizontal line. In the narrow shaded region to the very left, electromagnetism is weaker than gravity and therefore irrelevant.

More generally, the apparent coincidence of the habitable and observed values of some physical parameter can be taken as evidence for the existence of a larger ensemble, of which what we observe is merely one member among many (Carter 1973). Although the existence of other hotel rooms and solar systems is uncontroversial and observationally confirmed, that of parallel universes is not, since they cannot be observed. Yet if fine-tuning is observed, one can argue for their existence using the exact same logic as above. Indeed, there are numerous examples of fine tuning suggesting parallel universes with other physical constants, although the degree of fine tuning is still under active debate and should be clarified by additional calculations — see Rees (2002) and Davies (1982) for popular accounts and Barrow & Tipler (1986) for technical details.

For instance, if the electromagnetic force were weakened by a mere 4%, then the Sun would immediately explode (the diproton would have a bound state, which would increase the solar luminosity by a factor 10^{18}). If it were stronger, there would be fewer stable atoms. Indeed, most if not all the parameters affecting low-energy physics appear fine-tuned at some level, in the sense that changing them by modest amounts results in a qualitatively different universe.

If the weak interaction were substantially weaker, there

would be no hydrogen around, since it would have been converted to helium shortly after the Big Bang. If it were either much stronger or much weaker, the neutrinos from a supernova explosion would fail to blow away the outer parts of the star, and it is doubtful whether life-supporting heavy elements would ever be able to leave the stars where they were produced. If the protons were 0.2% heavier, they would decay into neutrons unable to hold onto electrons, so there would be no stable atoms around. If the proton-to-electron mass ratio were much smaller, there could be no stable stars, and if it were much larger, there could be no ordered structures like crystals and DNA molecules.

Fine-tuning discussions often turn heated when somebody mentions the “A-word”, *anthropic*. The author feels that discussions of the so-called anthropic principle have generated more heat than light, with many different definitions and interpretations of what it means. The author is not aware of anybody disagreeing with what might be termed MAP, the *minimalistic anthropic principle*:

- MAP: When testing fundamental theories with observational data, ignoring selection effects can give incorrect conclusions.

This is obvious from our examples above: if we neglected selection effects, we would be surprised to orbit a star as heavy as the Sun, since lighter and dimmer ones are much more abundant. Likewise, MAP says that the chaotic inflation model is *not* ruled out by the fact that we find ourselves living in the minuscule fraction of space where inflation has ended, since the inflating part is uninhabitable to us. Fortunately, selection effects cannot rescue all models, as pointed out a century ago by Boltzmann. If the universe were in classical thermal equilibrium (heat death), thermal fluctuations could still make atoms assemble at random to briefly create a self-aware observer like you once in a blue moon, so the fact that you exist right now does not rule out the heat death cosmological model. However, you should statistically expect to find the rest of the world in a high-entropy mess rather than in the ordered low-entropy state you observe, which rules out this model.

The standard model of particle physics has 28 of free parameters, and cosmology may introduce additional independent ones. If we really do live in a Level II multiverse, then for those parameters that vary between the parallel universes, we will never be able to predict our measured values from first principles. We can merely compute probability distributions for what we should expect to find, taking selection effects into account. We should expect to find everything that can vary across the ensemble to be as generic as is consistent with our existence. As detailed in Section VB, this issue of what is “generic” and, more specifically, how to compute probabilities in physics, is emerging as an embarrassingly thorny problem (see Section VB).

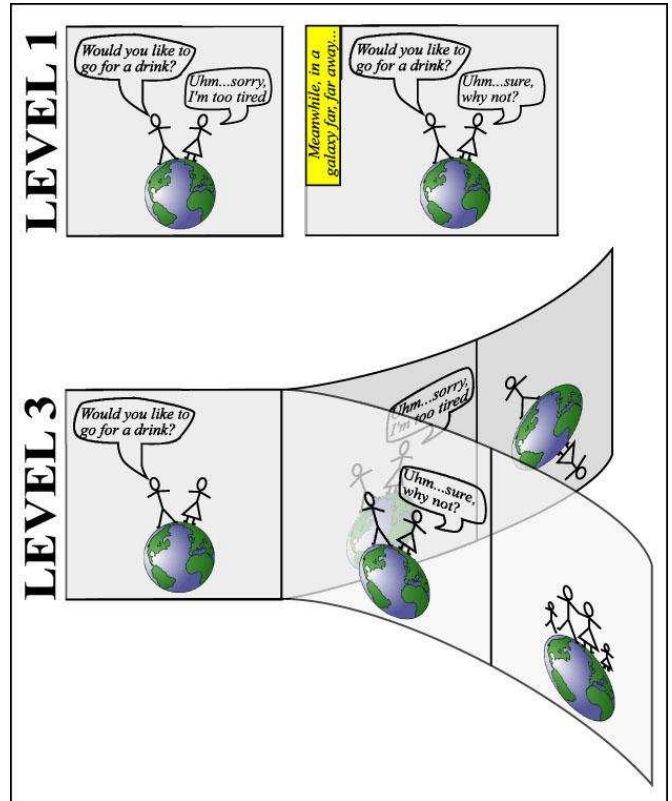


FIG. 5. Difference between Level I and Level III. Whereas Level I parallel universes are far away in space, those of Level III are even right here, with quantum events causing classical reality to split and diverge into parallel storylines. Yet Level III adds no new storylines beyond levels 1 or 2.

III. LEVEL III: THE MANY WORLDS OF QUANTUM PHYSICS

There may be a third type of parallel worlds that are not far away but in a sense right here. If the fundamental equations of physics are what mathematicians call *unitary*, as they so far appear to be, then the universe keeps branching into parallel universes as in the cartoon (Figure 5, bottom): whenever a quantum event appears to have a random outcome, all outcomes in fact occur, one in each branch. This is the Level III multiverse. Although more debated and controversial than Level I and Level II, we will see that, surprisingly, this level adds no new types of universes.

A. Evidence for Level III parallel universes

In the early 20th century, the theory of quantum mechanics revolutionized physics by explaining the atomic realm, with applications ranging from chemistry to nuclear reactions, lasers and semiconductors. Despite the obvious successes in its application, a heated debate ensued about its interpretation — a debate that still rages on. In quantum theory, the state of the universe is not given in classical terms such as the positions and velocities of all particles, but by a mathematical object called a wavefunction. According to the so-called Schrödinger equation, this state evolves deterministically over time in a fashion termed *unitary*, corresponding to a rotation in Hilbert space, the abstract infinite-dimensional space where the wavefunction lives. The sticky part is that there are perfectly legitimate wavefunctions corresponding to classically counterintuitive situations such as you being in two different places at once. Worse, the Schrödinger equation can evolve innocent classical states into such schizophrenic ones. As a baroque example, Schrödinger described the famous thought experiment where a nasty contraption kills a cat if a radioactive atom decays. Since the radioactive atom eventually enters a superposition of decayed and not decayed, it produces a cat which is both dead and alive in superposition.

In the 1920s, this weirdness was explained away by postulating that the wavefunction “collapsed” into some definite classical outcome whenever an observation was made, with probabilities given by the wavefunction. Einstein was unhappy about such intrinsic randomness in nature, which violated unitarity, insisting that “God doesn’t play dice”, and others complained that there was no equation specifying when this collapse occurred. In his 1957 Ph.D. thesis, Princeton student Hugh Everett III showed that this controversial collapse postulate was unnecessary. Quantum theory predicted that one classical reality would gradually split into superpositions of many (Figure 5). He showed that observers would subjectively experience this splitting merely as a slight ran-

domness, and indeed with probabilities in exact agreement with those from the old collapse postulate (de Witt 2003). This superposition of classical worlds is the Level III multiverse.

Everett’s work had left two crucial questions unanswered: first of all, if the world actually contains bizarre macrosuperpositions, then why don’t we perceive them? The answer came in 1970, when Dieter Zeh showed that the Schrödinger equation itself gives rise to a type of censorship effect (Zeh 1970). This effect became known as *decoherence*, and was worked out in great detail by Wojciech Zurek, Zeh and others over the following decades. Coherent quantum superpositions were found to persist only as long as they were kept secret from the rest of the world. A single collision with a snooping photon or air molecule is sufficient to ensure that our friends in Figure 5 can never be aware of their counterparts in the parallel storyline. A second unanswered question in the Everett picture was more subtle but equally important: what physical mechanism picks out approximately classical states (with each object in only one place, *etc.*) as special in the bewilderingly large Hilbert space? Decoherence answered this question as well, showing that classical states are simply those that are most robust against decoherence. In summary, decoherence both identifies the Level III parallel universes in Hilbert space and delimits them from one another. Decoherence is now quite uncontroversial and has been experimentally measured in a wide range of circumstances. Since decoherence for all practical purposes mimics wavefunction collapse, it has eliminated much of the original motivation for non-unitary quantum mechanics and made the Everett’s so-called many worlds interpretation increasingly popular. For details about these quantum issues, see Tegmark & Wheeler (2001) for a popular account and Giulini *et al.* (1996) for a technical review.

If the time-evolution of the wavefunction is unitary, then the Level III multiverse exists, so physicists have worked hard on testing this crucial assumption. So far, no departures from unitarity have been found. In the last few decades, remarkable experiments have confirmed unitarity for ever larger systems, including the hefty carbon-60 “Bucky Ball” atom and kilometer-size optical fiber systems. On the theoretical side, a leading argument against unitarity has involved possible destruction of information during the evaporation of black holes, suggesting that quantum-gravitational effects are non-unitary and collapse the wavefunction. However, a recent string theory breakthrough known as AdS/CFT correspondence has suggested that even quantum gravity is unitary, being mathematically equivalent to a lower-dimensional quantum field theory without gravity (Maldacena 2003).

B. What are Level III parallel universes like?

When discussing parallel universes, we need to distinguish between two different ways of viewing a physical theory: the outside view or *bird perspective* of a mathematician studying its mathematical fundamental equations and the inside view or *frog perspective* of an observer living in the world described by the equations^{***}. From the bird perspective, the Level III multiverse is simple: there is only one wavefunction, and it evolves smoothly and deterministically over time without any sort of splitting or parallelism. The abstract quantum world described by this evolving wavefunction contains within it a vast number of parallel classical storylines (see Figure 5), continuously splitting and merging, as well as a number of quantum phenomena that lack a classical description. From her frog perspective, however, each observer perceives only a tiny fraction of this full reality: she can only see her own Hubble volume (Level I) and decoherence prevents her from perceiving Level III parallel copies of herself. When she is asked a question, makes a snap decision and answers (Figure 5), quantum effects at the neuron level in her brain lead to multiple outcomes, and from the bird perspective, her single past branches into multiple futures. From their frog perspectives, however, each copy of her is unaware of the other copies, and she perceives this quantum branching as merely a slight randomness. Afterwards, there are for all practical purposes multiple copies of her that have the exact same memories up until the point when she answers the question.

^{***}Indeed, the standard mental picture of what the physical world is corresponds to a third intermediate viewpoint that could be termed the *consensus view*. From your subjectively perceived frog perspective, the world turns upside down when you stand on your head and disappears when you close your eyes, yet you subconsciously interpret your sensory inputs as though there is an external reality that is independent of your orientation, your location and your state of mind. It is striking that although this third view involves both censorship (like rejecting dreams), interpolation (as between eye-blinks) and extrapolation (say attributing existence to unseen cities) of your inside view, independent observers nonetheless appear to share this consensus view. Although the inside view looks black-and-white to a cat, iridescent to a bird seeing four primary colors, and still more different to a bee seeing polarized light, a bat using sonar, a blind person with keener touch and hearing, or the latest overpriced robotic vacuum cleaner, all agree on whether the door is open. The key current challenge in physics is deriving this semiclassical consensus view from the fundamental equations specifying the bird perspective. In my opinion, this means that although understanding the detailed nature of human consciousness is an important challenge in its own right, it is *not* necessary for a fundamental theory of physics.

C. How many different parallel universes are there?

As strange as this may sound, Figure 5 illustrates that this exact same situation occurs even in the Level I multiverse, the only difference being where her copies reside (elsewhere in good old three-dimensional space as opposed to elsewhere in infinite-dimensional Hilbert space, in other quantum branches). In this sense, Level III is no stranger than Level I. Indeed, if physics is unitary, then the quantum fluctuations during inflation did not generate unique initial conditions through a random process, but rather generated a quantum superposition of all possible initial conditions simultaneously, after which decoherence caused these fluctuations to behave essentially classically in separate quantum branches. The ergodic nature of these quantum fluctuations (Section I B) therefore implies that the distribution of outcomes in a given Hubble volume at Level III (between different quantum branches as in Fig 3) is identical to the distribution that you get by sampling different Hubble volumes within a single quantum branch (Level I). If physical constants, spacetime dimensionality *etc.* can vary as in Level II, then they too will vary between parallel quantum branches at Level III. The reason for this is that if physics is unitary, then the process of spontaneous symmetry breaking will not produce a unique (albeit random) outcome, but rather a superposition of all outcomes that rapidly decoheres into for all practical purposes separate Level III branches. In short, the Level III multiverse, if it exists, adds nothing new beyond Level I and Level II — just more indistinguishable copies of the same universes, the same old storylines playing out again and again in other quantum branches. Postulating a yet unseen non-unitary effect to get rid of the Level III multiverse, with Ockham’s Razor in mind, therefore would not make Ockham any happier.

The passionate debate about Everett’s parallel universes that has raged on for decades therefore seems to be ending in a grand anticlimax, with the discovery of a less controversial multiverse that is just as large. This is reminiscent of the famous Shapley-Curtis debate of the 1920s about whether there were really a multitude of galaxies (parallel universes by the standards of the time) or just one, a storm in a teacup now that research has moved on to other galaxy clusters, superclusters and even Hubble volumes. In hindsight, both the Shapley-Curtis and Everett controversies seem positively quaint, reflecting our instinctive reluctance to expand our horizons.

A common objection is that repeated branching would exponentially increase the number of universes over time. However, the number of universes N may well stay constant. By the number of “universes” N , we mean the number that are indistinguishable from the frog perspective (from the bird perspective, there is of course just one) at a given instant, *i.e.*, the number of macroscop-

ically different Hubble volumes. Although there is obviously a vast number of them (imagine moving planets to random new locations, imagine having married someone else, *etc.*), the number N is clearly finite — even if we pedantically distinguish Hubble volumes at the quantum level to be overly conservative, there are “only” about $10^{10^{15}}$ with temperature below 10^8K as detailed above.^{†††} The smooth unitary evolution of the wavefunction in the bird perspective corresponds to a never-ending sliding between these N classical universe snapshots from the frog perspective of an observer. Now you’re in universe A, the one where you’re reading this sentence. Now you’re in universe B, the one where you’re reading this other sentence. Put differently, universe B has an observer identical to one in universe A, except with an extra instant of memories. In Figure 5, our observer first finds herself in the universe described by the left panel, but now there are two different universes smoothly connecting to it like B did to A, and in both of these, she will be unaware of the other one. Imagine drawing a separate dot corresponding to each possible universe and drawing arrows indicating which ones connect to which in the frog perspective. A dot could lead uniquely to one other dot or to several, as above. Likewise, several dots could lead to one and the same dot, since there could be many dif-

^{†††}For the technical reader, could the grand superposition of the universal wavefunctional involve other interesting states besides the semiclassical ones? Specifically, the semiclassical states (corresponding to what we termed the consensus view) are those that are maximally robust towards decoherence (Zurek 2003), so if we project out the component of the wavefunctional that is spanned by these states, what remains? We can make a hand-waving argument that all that remains is a rather uninteresting high-energy mess which will be devoid of observers and rapidly expand or collapse. Let us consider the special case of the electromagnetic field. In many circumstances (Anglin & Zurek 1996), its semiclassical states can be shown to be generalized coherent states, which have infinite-dimensional Gaussian Wigner functions with characteristic widths no narrower than the those corresponding to the local temperature. Such functions form a well-conditioned basis for all states whose wavefunction is correspondingly smooth, *i.e.*, lacking violent high-energy fluctuations. This is illustrated in Figure 6 for the simple case of a 1-dimensional quantum particle: the wavefunction $\psi(x)$ can be written as a superposition of a low energy (low-pass filtered) and a high-energy (high-pass filtered) part, and the former can be decomposed as the convolution of a smooth function with a Gaussian, *i.e.*, as a superposition of coherent states with Gaussian wavepackets. Decoherence rapidly makes the macroscopically distinct semiclassical states of the electromagnetic field for all practical purposes separate both from each other and from the high-energy mess. The high-energy component may well be typical of the early universe that we evolved from.

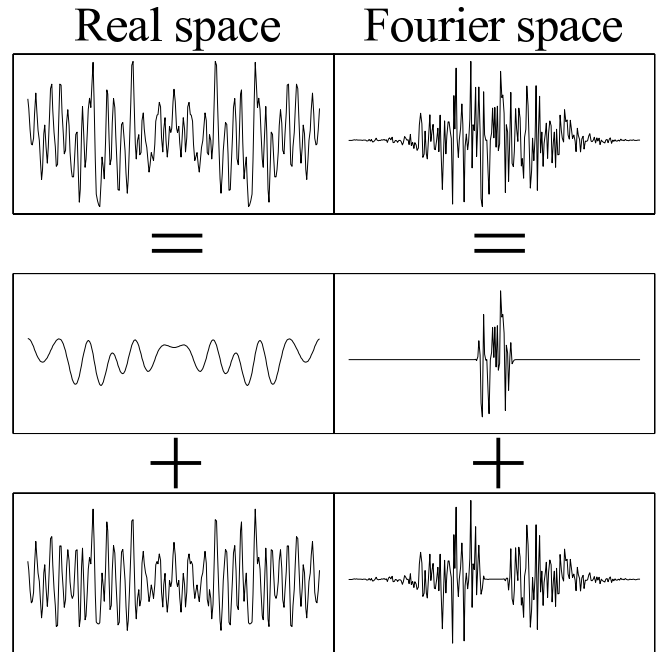


FIG. 6. Schematic illustration (see footnote) of how a wavefunctional of the Level 3 multiverse (top row for simple 1-dimensional Hilbert space) can be decomposed as a superposition of semiclassical worlds (generalized coherent states; middle row) and a high-energy mess (bottom row).

ferent ways in which certain situations could have come about. The Level III multiverse thus involves not only splitting branches but merging branches as well.

Ergodicity implies that the quantum state of the Level III multiverse is invariant under spatial translations, which is a unitary operation just as time translation. If it is invariant under time-translation as well (this can be arranged by constructing a superposition of an infinite set of quantum states that are all different time translations of one and the same state, so that a Big Bang happens at different times in different quantum branches), then the number of universes would automatically stay exactly constant. All possible universe snapshots would exist at every instant, and the passage of time would just be in the eye of the beholder — an idea explored in the sci-fi novel “Permutation City” (Egan 1995) and developed by Deutsch (1997), Barbour (2001) and others.

D. Two world views

The debate over how classical mechanics emerges from quantum mechanics continues, and the decoherence discovery has shown that there is a lot more to it than just letting Planck’s constant \hbar shrink to zero. Yet as Figure 7 illustrates, this is just a small piece of a larger puzzle. Indeed, the endless debate over the interpretation of quantum mechanics — and even the broader issue of parallel universes — is in a sense the tip of an iceberg. In the

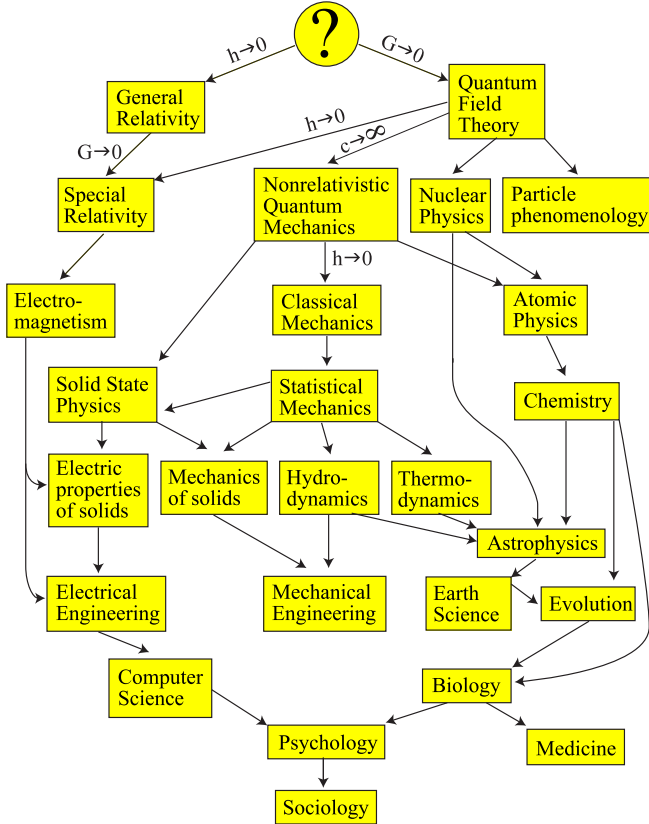


FIG. 7. Theories can be crudely organized into a family tree where each might, at least in principle, be derivable from more fundamental ones above it. For example, classical mechanics can be obtained from special relativity in the approximation that the speed of light c is infinite. Most of the arrows are less well understood. All these theories have two components: mathematical equations and words that explain how they are connected to what we observe. At each level in the hierarchy of theories, new words (e.g., protons, atoms, cells, organisms, cultures) are introduced because they are convenient, capturing the essence of what is going on without recourse to the more fundamental theory above it. It is important to remember, however, that it is we humans who introduce these concepts and the words for them: in principle, everything could have been derived from the fundamental theory at the top of the tree, although such an extreme reductionist approach would of course be useless in practice. Crudely speaking, the ratio of equations to words decreases as we move down the tree, dropping near zero for highly applied fields such as medicine and sociology. In contrast, theories near the top are highly mathematical, and physicists are still struggling to understand the concepts, if any, in terms of which we can understand them. The Holy Grail of physics is to find what is jocularly referred to as a “Theory of Everything”, or TOE, from which all else can be derived. If such a theory exists at all, it should replace the big question mark at the top of the theory tree. Everybody knows that something is missing here, since we lack a consistent theory unifying gravity with quantum mechanics.

Sci-Fi spoof “Hitchhiker’s Guide to the Galaxy”, the answer is discovered to be “42”, and the hard part is finding the real question. Questions about parallel universes may seem to be just about as deep as queries about reality can get. Yet there is a still deeper underlying question: there are two tenable but diametrically opposed paradigms regarding physical reality and the status of mathematics, a dichotomy that arguably goes as far back as Plato and Aristotle, and the question is which one is correct.

- **ARISTOTELIAN PARADIGM:** The subjectively perceived frog perspective is physically real, and the bird perspective and all its mathematical language is merely a useful approximation.
- **PLATONIC PARADIGM:** The bird perspective (the mathematical structure) is physically real, and the frog perspective and all the human language we use to describe it is merely a useful approximation for describing our subjective perceptions.

What is more basic — the frog perspective or the bird perspective? What is more basic — human language or mathematical language? Your answer will determine how you feel about parallel universes. If you prefer the Platonic paradigm, you should find multiverses natural, since our feeling that say the Level III multiverse is “weird” merely reflects that the frog and bird perspectives are extremely different. We break the symmetry by calling the latter weird because we were all indoctrinated with the Aristotelian paradigm as children, long before we even heard of mathematics - the Platonic view is an acquired taste!

In the second (Platonic) case, all of physics is ultimately a mathematics problem, since an infinitely intelligent mathematician given the fundamental equations of the cosmos could in principle *compute* the frog perspective, *i.e.*, compute what self-aware observers the universe would contain, what they would perceive, and what language they would invent to describe their perceptions to one another. In other words, there is a “Theory of Everything” (TOE) at the top of the tree in Figure 7 whose axioms are purely mathematical, since postulates in English regarding interpretation would be derivable and thus redundant. In the Aristotelian paradigm, on the other hand, there can never be a TOE, since one is ultimately just explaining certain verbal statements by other verbal statements — this is known as the infinite regress problem (Nozick 1981).

IV. LEVEL IV: OTHER MATHEMATICAL STRUCTURES

Suppose you buy the Platonist paradigm and believe that there really is a TOE at the top of Figure 7 — and

that we simply have not found the correct equations yet. Then an embarrassing question remains, as emphasized by John Archibald Wheeler: *Why these particular equations, not others?* Let us now explore the idea of mathematical democracy, whereby universes governed by other equations are equally real. This is the Level IV multiverse. First we need to digest two other ideas, however: the concept of a mathematical structure, and the notion that the physical world may be one.

A. What is a mathematical structure?

Many of us think of mathematics as a bag of tricks that we learned in school for manipulating numbers. Yet most mathematicians have a very different view of their field. They study more abstract objects such as functions, sets, spaces and operators and try to prove theorems about the relations between them. Indeed, some modern mathematics papers are so abstract that the only numbers you will find in them are the page numbers! What does a dodecahedron have in common with a set of complex numbers? Despite the plethora of mathematical structures with intimidating names like orbifolds and Killing fields, a striking underlying unity that has emerged in the last century: *all* mathematical structures are just special cases of one and the same thing: so-called formal systems. A formal system consists of abstract symbols and rules for manipulating them, specifying how new strings of symbols referred to as theorems can be derived from given ones referred to as axioms. This historical development represented a form of deconstructionism, since it stripped away all meaning and interpretation that had traditionally been given to mathematical structures and distilled out only the abstract relations capturing their very essence. As a result, computers can now prove theorems about geometry without having any physical intuition whatsoever about what space is like.

Figure 8 shows some of the most basic mathematical structures and their interrelations. Although this family tree probably extends indefinitely, it illustrates that there is nothing fuzzy about mathematical structures. They are “out there” in the sense that mathematicians discover them rather than create them, and that contemplative alien civilizations would find the same structures (a theorem is true regardless of whether it is proven by a human, a computer or an alien).

B. The possibility that the physical world is a mathematical structure

Let us now digest the idea that physical world (specifically, the Level III multiverse) *is* a mathematical structure. Although traditionally taken for granted by many

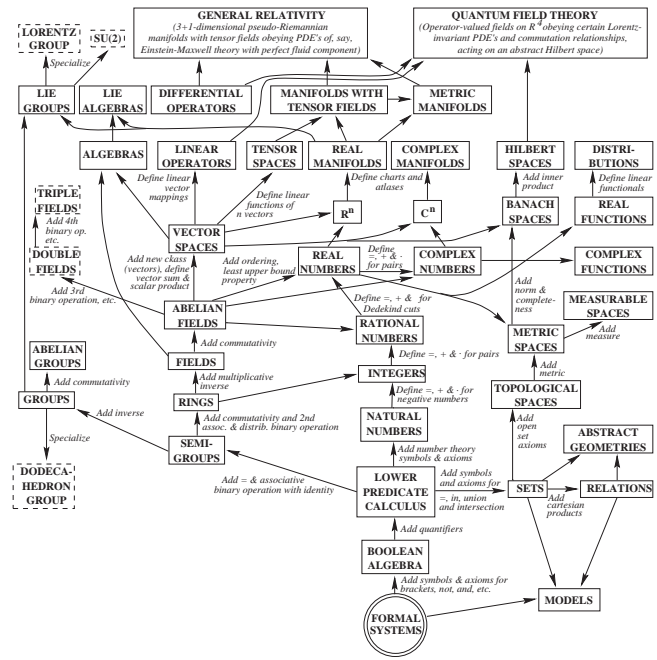


FIG. 8. Relationships between various basic mathematical structures (Tegmark 1998). The arrows generally indicate addition of new symbols and/or axioms. Arrows that meet indicate the combination of structures — for instance, an algebra is a vector space that is also a ring, and a Lie group is a group that is also a manifold. The full tree is probably infinite in extent — the figure shows merely a small sample near the bottom.

theoretical physicists, this is a deep and far-reaching notion. It means that mathematical equations describe not merely some limited aspects of the physical world, but *all* aspects of it. It means that there is some mathematical structure that is what mathematicians call *isomorphic* (and hence equivalent) to our physical world, with each physical entity having a unique counterpart in the mathematical structure and vice versa. Let us consider some examples.

A century ago, when classical physics still reigned supreme, many scientists believed that physical space was isomorphic to the mathematical structure known as \mathbf{R}^3 : three-dimensional Euclidean space. Moreover, some thought that all forms of matter in the universe corresponded to various classical *fields*: the electric field, the magnetic field and perhaps a few undiscovered ones, mathematically corresponding to functions on \mathbf{R}^3 (a handful of numbers at each point in space). In this view (later proven incorrect), dense clumps of matter like atoms were simply regions in space where some fields were strong (where some numbers were large). These fields evolved deterministically over time according to some partial differential equations, and observers perceived this as things moving around and events taking place. Could, then, fields in three-dimensional space be the mathematical structure corresponding to the uni-

verse? No, since a mathematical structure cannot change — it is an abstract, immutable entity existing outside of space and time. Our familiar frog perspective of a three-dimensional space where events unfold is equivalent, from the bird perspective, to a four-dimensional spacetime where all of history is contained, so the mathematical structure would be fields in four-dimensional space. In other words, if history were a movie, the mathematical structure would not correspond to a single frame of it, but to the entire videotape.

Given a mathematical structure, we will say that it has *physical existence* if any self-aware substructure (SAS) within it subjectively, from its frog perspective, perceives itself as living in a physically real world. What would, mathematically, such an SAS be like? In the classical physics example above, an SAS such as you would be a tube through spacetime, a thick version of what Einstein referred to as a world-line. The location of the tube would specify your position in space at different times. Within the tube, the fields would exhibit certain complex behavior, corresponding to storing and processing information about the field-values in the surroundings, and at each position along the tube, these processes would give rise to the familiar but mysterious sensation of self-awareness. From its frog perspective, the SAS would perceive this one-dimensional string of perceptions along the tube as passage of time.

Although our example illustrates the idea of how our physical world can *be* a mathematical structure, this particular mathematical structure (fields in four-dimensional space) is now known to be the wrong one. After realizing that spacetime could be curved, Einstein doggedly searched for a so-called unified field theory where the universe was what mathematicians call a 3+1-dimensional pseudo-Riemannian manifold with tensor fields, but this failed to account for the observed behavior of atoms. According to quantum field theory, the modern synthesis of special relativity theory and quantum theory, the universe (in this case the Level III multiverse) is a mathematical structure known as an algebra of operator-valued fields. Here the question of what constitutes an SAS is more subtle (Tegmark 2000). However, this fails to describe black hole evaporation, the first instance of the Big Bang and other quantum gravity phenomena, so the true mathematical structure isomorphic to our universe, if it exists, has not yet been found.

C. Mathematical democracy

Now suppose that our physical world really is a mathematical structure, and that you are an SAS within it. This means that in the Mathematics tree of Figure 8, one of the boxes is our universe. (The full tree is probably infinite in extent, so our particular box is not one of the few boxes from the bottom of the tree that are shown.)

In other words, this particular mathematical structure enjoys not only mathematical existence, but physical existence as well. What about all the other boxes in the tree? Do they too enjoy physical existence? If not, there would be a fundamental, unexplained ontological asymmetry built into the very heart of reality, splitting mathematical structures into two classes: those with and without physical existence. As a way out of this philosophical conundrum, I have suggested (Tegmark 1998) that complete mathematical democracy holds: that mathematical existence and physical existence are equivalent, so that *all* mathematical structures exist physically as well. This is the Level IV multiverse. It can be viewed as a form of radical Platonism, asserting that the mathematical structures in Plato’s *realm of ideas*, the *Mindscape* of Rucker (1982), exist “out there” in a physical sense (Davies 1993), casting the so-called modal realism theory of David Lewis (1986) in mathematical terms akin to what Barrow (1991; 1992) refers to as “ π in the sky”. If this theory is correct, then since it has no free parameters, all properties of all parallel universes (including the subjective perceptions of SASs in them) could in principle be derived by an infinitely intelligent mathematician.

D. Evidence for a Level IV multiverse

We have described the four levels of parallel universes in order of increasing speculativeness, so why should we believe in Level IV? Logically, it rests on two separate assumptions:

- **Assumption 1:** That the physical world (specifically our level III multiverse) is a mathematical structure
- **Assumption 2:** Mathematical democracy: that all mathematical structures exist “out there” in the same sense

In a famous essay, Wigner (1967) argued that “the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious”, and that “there is no rational explanation for it”. This argument can be taken as support for assumption 1: here the utility of mathematics for describing the physical world is a natural consequence of the fact that the latter *is* a mathematical structure, and we are simply uncovering this bit by bit. The various approximations that constitute our current physics theories are successful because simple mathematical structures can provide good approximations of how a SAS will perceive more complex mathematical structures. In other words, our successful theories are not mathematics approximating physics, but mathematics approximating mathematics. Wigner’s observation is unlikely to be based on fluke coincidences, since far more mathematical regularity in nature has been

discovered in the decades since he made it, including the standard model of particle physics.

A second argument supporting assumption 1 is that abstract mathematics is so general that *any* TOE that is definable in purely formal terms (independent of vague human terminology) is also a mathematical structure. For instance, a TOE involving a set of different types of entities (denoted by words, say) and relations between them (denoted by additional words) is nothing but what mathematicians call a set-theoretical model, and one can generally find a formal system that it is a model of.

This argument also makes assumption 2 more appealing, since it implies that *any* conceivable parallel universe theory can be described at Level IV. The Level IV multiverse, termed the “ultimate Ensemble theory” in Tegmark (1997) since it subsumes all other ensembles, therefore brings closure to the hierarchy of multiverses, and there cannot be say a Level V. Considering an ensemble of mathematical structures does not add anything new, since this is still just another mathematical structure. What about the frequently discussed notion that the universe is a computer simulation? This idea occurs frequently in science fiction and has been substantially elaborated (*e.g.*, Schmidhuber 1997; Wolfram 2002). The information content (memory state) of a digital computer is a string of bits, say “1001011100111001...” of great but finite length, equivalent to some large but finite integer n written in binary. The information processing of a computer is a deterministic rule for changing each memory state into another (applied over and over again), so mathematically, it is simply a function f mapping the integers onto themselves that gets iterated: $n \mapsto f(n) \mapsto f(f(n)) \mapsto \dots$. In other words, even the most sophisticated computer simulation is just yet another special case of a mathematical structure, and is already included in the Level IV multiverse. (Incidentally, iterating continuous functions rather than integer-valued ones can give rise to fractals.)

Another appealing feature of assumption 2 is that it provides the only answer so far to Wheeler’s question: *Why these particular equations, not others?* Having universes dance to the tune of all possible equations also resolves the fine-tuning problem of Section II C once and for all, even at the fundamental equation level: although many if not most mathematical structures are likely to be dead and devoid of SASs, failing to provide the complexity, stability and predictability that SASs require, we of course expect to find with 100% probability that we inhabit a mathematical structure capable of supporting life. Because of this selection effect, the answer to the question “what is it that breathes fire into the equations and makes a universe for them to describe?” (Hawking 1993) would then be “you, the SAS”.

E. What are Level IV parallel universes like?

The way we use, test and potentially rule out any theory is to compute probability distributions for our future perceptions given our past perceptions and to compare these predictions with our observed outcome. In a multiverse theory, there is typically more than one SAS that has experienced a past life identical to yours, so there is no way to determine which one is you. To make predictions, you therefore have to compute what fractions of them will perceive what in the future, which leads to the following predictions:

- **Prediction 1:** The mathematical structure describing our world is the most generic one that is consistent with our observations.
- **Prediction 2:** Our future observations are the most generic ones that are consistent with our past observations.
- **Prediction 3:** Our past observations are the most generic ones that are consistent with our existence.

We will return to the problem of what “generic” means in

secMeasureSec (the measure problem). However, one striking feature of mathematical structures, discussed in detail in Tegmark (1997), is that the sort of symmetry and invariance properties that are responsible for the simplicity and orderliness of our universe tend to be generic, more the rule than the exception — mathematical structures tend to have them by default, and complicated additional axioms *etc.* must be added to make them go away. In other words, because of both this and selection effects, we should not necessarily expect life in the Level IV multiverse to be a disordered mess.

V. DISCUSSION

We have surveyed scientific theories of parallel universes, and found that they naturally form a four-level hierarchy of multiverses (Figure 1) allowing progressively greater differences from our own universe:

- Level I: Other Hubble volumes have different initial conditions
- Level II: Other post-inflation bubbles may have different effective laws of physics (constants, dimensionality, particle content)
- Level III: Other branches of the quantum wavefunction add nothing qualitatively new
- Level IV: Other mathematical structures have different fundamental equations of physics

Whereas the Level I universes join seamlessly, there are clear demarcations between those within levels II and III caused by inflating space and decoherence, respectively. The level IV universes are completely separate and need to be considered together only for predicting your future, since “you” may exist in more than one of them.

Although it was Level I that got Giordano Bruno in trouble with the inquisition, few astronomers today would suggest that space ends abruptly at the edge of the observable universe. It is ironic and perhaps due to historic coincidence that Level III is the one that has drawn the most fire in the past decades, since it is the only one that adds no qualitatively new types of universes.

A. Future prospects

There are ample future prospects for testing and perhaps ruling out these multiverse theories. In the coming decade, dramatically improved cosmological measurements of the microwave background radiation, the large-scale matter distribution, *etc.*, will test Level I by further constraining the curvature and topology of space and will test level II by providing stringent tests of inflation. Progress in both astrophysics and high-energy physics should also clarify the extent to which various physical constants are fine-tuned, thereby weakening or strengthening the case for Level II. If the current world-wide effort to build quantum computers succeeds, it will provide further evidence for Level III, since they would, in essence, be exploiting the parallelism of the Level III multiverse for parallel computation (Deutsch 1997). Conversely, experimental evidence of unitarity violation would rule out Level III. Finally, success or failure in the grand challenge of modern physics, unifying general relativity and quantum field theory, will shed more light on Level IV. Either we will eventually find a mathematical structure matching our universe, or we will bump up against a limit to the unreasonable effectiveness of mathematics and have to abandon Level IV.

B. The measure problem

There are also interesting theoretical issues to resolve within the multiverse theories, first and foremost the *measure problem*. As multiverse theories gain credence, the sticky issue of how to compute probabilities in physics is growing from a minor nuisance into a major embarrassment. The reason why probabilities become so important is that if there are indeed many copies of “you” with identical past lives and memories, you could not compute your own future even if you had complete knowledge of the entire state of the multiverse. This is because there is no way for you to determine which of these copies is “you” (they all feel that they are). All you can predict

is therefore probabilities for what you will observe, corresponding to the fractions of these observers that experience different things. Unfortunately, computing what fraction of the infinitely many observers perceive what is very subtle, since the answer depends on the order in which you count them! The fraction of the integers that are even is 50% if you order them 1, 2, 3, 4..., but approaches 100% if you order them alphabetically the way your word processor would (1, 10, 100, 1000, ...). When observers reside in disconnected universes, there is no obviously natural way in which to order them, and one must sample from the different universes with some statistical weights referred to by mathematicians as a “measure”. This problem crops up in a mild and treatable manner in Level I, becomes severe at Level II, has caused much debate within the context of extracting quantum probabilities in Level III (de Witt 2003), and is horrendous at Level IV. At Level II, for instance, Vilenkin and others have published predictions for the probability distributions of various cosmological parameters by arguing that different parallel universes that have inflated by different amounts should be given statistical weights proportional to their volume (*e.g.*, Garriga & Vilenkin 2001a). On the other hand, any mathematician will tell you that $2 \times \infty = \infty$, so that there is no objective sense in which an infinite universe that has expanded by a factor of two has gotten larger. Indeed, an exponentially inflating universe has what mathematicians call a time-like Killing vector, which means that it is time-translationally invariant and hence unchanging from a mathematical point of view. Moreover, a flat universe with finite volume and the topology of a torus is equivalent to a perfectly periodic universe with infinite volume, both from the mathematical bird perspective and from the frog perspective of an observer within it, so why should its infinitely smaller volume give it zero statistical weight? Since Hubble volumes start repeating even in the Level I multiverse (albeit in a random order, not periodically) after about $10^{10^{115}}$ meters, should infinite space really be given more statistical weight than a finite region of that size? This problem must be solved to observationally test models of stochastic inflation. If you thought that was bad, consider the problem of assigning statistical weights to different mathematical structures at Level IV. The fact that our universe seems relatively simple has led many people to suggest that the correct measure somehow involves complexity. For instance, one could reward simplicity by weighting each mathematical structure by 2^{-n} , where n is its algorithmic information content measured in bits, defined as the length of the shortest bit string (computer program, say) that would specify it (Chaitin 1987). This would correspond to equal weights for all infinite bit strings (each representable as a real number like .101011101...), not for all mathematical structures. If there is such an exponential penalty for high complex-

ity, we should probably expect to find ourselves inhabiting one of the simplest mathematical structures complex enough to contain observers. However, the algorithmic complexity depends on how structures are mapped to bit strings (Chaitin 1987; Deutsch 2003), and it far from obvious whether there exists a most natural definition that reality might subscribe to.

C. The pros and cons of parallel universes

So should you believe in parallel universes? Let us conclude with a brief discussion of arguments pro and con. First of all, we have seen that this is not a yes/no question — rather, the most interesting issue is whether there are 0, 1, 2, 3 or 4 levels of multiverses. Figure 1 summarizes evidence for the different levels. Cosmology observations support Level I by pointing to a flat infinite space with ergodic matter distribution, and Level I plus inflation elegantly eliminates the initial condition problem. Level II is supported by the success of inflation theory in explaining cosmological observations, and it can explain apparent fine-tuning of physical parameters. Level III is supported by both experimental and theoretical evidence for unitarity, and explains the apparent quantum randomness that bothered Einstein so much without abandoning causality from the bird perspective. Level IV explains Wigner’s unreasonable effectiveness of mathematics for describing physics and answers the question “why these equations, not others?”.

The principal arguments against parallel universes are that they are wasteful and weird, so let us consider these two objections in turn. The first argument is that multiverse theories are vulnerable to Ockham’s razor, since they postulate the existence of other worlds that we can never observe. Why should nature be so ontologically wasteful and indulge in such opulence as to contain an infinity of different worlds? Intriguingly, this argument can be turned around to argue *for* a multiverse. When we feel that nature is wasteful, what precisely are we disturbed about her wasting? Certainly not “space”, since the standard flat universe model with its infinite volume draws no such objections. Certainly not “mass” or “atoms” either, for the same reason — once you have wasted an infinite amount of something, who cares if you waste some more? Rather, it is probably the apparent reduction in simplicity that appears disturbing, the quantity of information necessary to specify all these unseen worlds. However, as is discussed in more detail in Tegmark (1996), an entire ensemble is often much simpler than one of its members. For instance, the algorithmic information content of a generic integer n is of order $\log_2 n$ (Chaitin 1987), the number of bits required to write it out in binary. Nonetheless, the set of all integers 1, 2, 3, ... can be generated by quite a trivial computer program, so the algorithmic complexity of the whole set is smaller than that of

a generic member. Similarly, the set of all perfect fluid solutions to the Einstein field equations has a smaller algorithmic complexity than a generic particular solution, since the former is specified simply by giving a few equations and the latter requires the specification of vast amounts of initial data on some hypersurface. Loosely speaking, the apparent information content rises when we restrict our attention to one particular element in an ensemble, thus losing the symmetry and simplicity that was inherent in the totality of all elements taken together. In this sense, the higher level multiverses have less algorithmic complexity. Going from our universe to the Level I multiverse eliminates the need to specify initial conditions, upgrading to Level II eliminates the need to specify physical constants and the Level IV multiverse of all mathematical structures has essentially no algorithmic complexity at all. Since it is merely in the frog perspective, in the subjective perceptions of observers, that this opulence of information and complexity is really there, a multiverse theory is arguably more economical than one endowing only a single ensemble element with physical existence (Tegmark 1996).

The second common complaint about multiverses is that they are weird. This objection is aesthetic rather than scientific, and as mentioned above, really only makes sense in the Aristotelian world view. In the Platonic paradigm, one might expect observers to complain that the correct TOE was weird if the bird perspective was sufficiently different from the frog perspective, and there is every indication that this is the case for us. The perceived weirdness is hardly surprising, since evolution provided us with intuition only for the everyday physics that had survival value for our distant ancestors. Thanks to clever inventions, we have glimpsed slightly more than the frog perspective of our normal inside view, and sure enough, we have encountered bizarre phenomena whenever departing from human scales in any way: at high speeds (time slows down), on small scales (quantum particles can be at several places at once), on large scales (black holes), at low temperatures (liquid Helium can flow upward), at high temperatures (colliding particles can change identity), *etc.* As a result, physicists have by and large already accepted that the frog and bird perspectives are very different. A prevalent modern view of quantum field theory is that the standard model is merely an effective theory, a low-energy limit of a yet to be discovered theory that is even more removed from our cozy classical concepts (involving strings in 10 dimensions, say). Many experimentalists are becoming blasé about producing so many “weird” (but perfectly repeatable) experimental results, and simply accept that the world is a weirder place than we thought it was and get on with their calculations.

We have seen that a common feature of all four multiverse levels is that the simplest and arguably most elegant theory involves parallel universes by default, and

that one needs to complicate the theory by adding experimentally unsupported processes and ad hoc postulates (finite space, wavefunction collapse, ontological asymmetry, *etc.*) to explain away the parallel universes. Our aesthetic judgement therefore comes down to what we find more wasteful and inelegant: many worlds or many words. Perhaps we will gradually get more used to the weird ways of our cosmos, and even find its strangeness to be part of its charm.

Acknowledgements: The author wishes to thank Anthony Aguirre, Aaron Classens, Angelica de Oliveira-Costa, George Musser, David Raub, Martin Rees, Harold Shapiro and Alex Vilenkin for stimulating discussions. This work was supported by NSF grants AST-0071213 & AST-0134999, NASA grants NAG5-9194 & NAG5-11099, a fellowship from the David and Lucile Packard Foundation and a Cottrell Scholarship from Research Corporation.

VI. REFERENCES

- Anglin, J. R., & Zurek, W. H. 1996, *Phys. Rev. D*, **53**, 7327
- Barbour J B, 2001 (The End of Time: Oxford) Univ. Press;Oxford
- Barrow, J. D. 1991, *Theories of Everything* (Ballantine: New York)
- Barrow, J. D. 1992, *Pi in the Sky* (Clarendon: Oxford)
- Barrow, J. D., & Tipler, F. J. 1986, *The Anthropic Cosmological Principle* (Clarendon: Oxford)
- Brundrit, G. B. 1979, *Q. J. Royal Astr. Soc.*, **20**, 37
- Bucher, M. A., & Spergel, D. N. 1999, *Sci. Am.* **1/1999**
- Carter, B. 1974, in *IAU Symposium 63*, ed. Longair, S. (Reidel: Dordrecht)
- Chaitin, G. J. 1987, *Algorithmic Information Theory* (Cambridge U. P: Cambridge)
- Davies, P. C. W. 1982, *The Accidental Universe* (Cambridge U. P: Cambridge)
- Davies, P. 1993, *The Mind of God* (Touchstone: New York)
- Davies, P. 1996, *Are We Alone?* (Basic Books: New York)
- Deutsch, D. 1997, *The Fabric of Reality* (Allen Lane: New York)
- Deutsch, D. 2003, in *Science and Ultimate Reality: From Quantum to Cosmos*, ed. Barrow, J. D., Davies, P. C. W., & Harper, C. L. (Cambridge Univ. Press: Cambridge)
- de Witt, B. 2003, in *Science and Ultimate Reality: From Quantum to Cosmos*, ed. Barrow, J. D., Davies, P. C. W., & Harper, C. L. (Cambridge Univ. Press: Cambridge)
- Egan, G. 1995, *Permutation City* (Harper: New York)
- Garriga, J., & Vilenkin, A. 2001a, *Phys. Rev. D*, **64**, 023507
- Garriga, J., & Vilenkin, A. 2001b, *Phys. Rev. D*, **64**, 043511.
- Giulini, D., Joos, E., Kiefer, C., Kupsch, J., Stamatescu, I. O., & Zeh, H. D. 1996, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Berlin: Springer)
- Guth, A., & Steinhardt, P. J. 1984, *Sci. Am.*, **250**, 116
- Hawking, S. 1993, *A Brief History of Time* (Touchstone: New York)
- Lewis, D. 1986, *On the Plurality of Worlds* (Blackwell: Oxford)
- Linde, A. 1994, *Sci. Am.*, **271**, 32
- Maldacena, J. 2003, in *Science and Ultimate Reality: From Quantum to Cosmos*, ed. Barrow, J. D., Davies, P. C. W., & Harper, C. L. (Cambridge Univ. Press: Cambridge)
- Nozick, R. 1981, *Philosophical Explanations* (Harvard Univ. Press: Cambridge)
- Rees, M. J. 2002, *Our Cosmic Habitat* (Princeton Univ. Press: Princeton)
- Rucker, R. 1982, *Infinity and the Mind* (Birkhauser: Boston)
- Schmidhuber, J. 1997, *A Computer Scientist's View of Life, the Universe, and Everything*, in *Foundations of Computer Science: Potential-Theory-Cognition, Lecture Notes in Computer Science*, C. Freksa, ed., (Springer: Berlin), <http://www.idsia.ch/~juergen/everything/html.html>
- Smolin, L. 1997, *The Life of the Cosmos* (Oxford Univ. Press: Oxford)
- Steinhardt, P. J., & Turok, N. 2002, *Science*, **296**, 1436
- Tegmark, M. 1996, *Found. Phys. Lett.*, **9**, 25
- Tegmark, M. 1997, *Class. Quant. Grav.*, **14**, L69
- Tegmark, M. 1998, *Ann. Phys.*, **270**, 1
- Tegmark, M. 2000, *Phys. Rev. E*, **61**, 4194
- Tegmark, M. 2002, *Science*, **296**, 1427
- Tegmark, M., & Wheeler, J. A. 2001, *Sci.Am.*, **2/2001**, 68-75
- Wigner, E. P. 1967, *Symmetries and Reflections* (MIT Press: Cambridge)
- Wolfram, S. 2002, *A New Kind of Science* (Wolfram Media: New York)
- Zeh, H. D. 1970, *Found. Phys.*, **1**, 69
- Zurek, W. 2003, in *Science and Ultimate Reality: From Quantum to Cosmos*, ed. Barrow, J. D., Davies, P. C. W., & Harper, C. L. (Cambridge Univ. Press: Cambridge)